

The Anomalous Magnetic Moment of the Muon

W. Morse - BNL

Physics

- I'm not trying to “teach physics” in this talk.
- You have many great professors who will do that.
- I'm just trying to convince you that physics is interesting.
- If you have a question, “raise your hand” or use the “chat”.
- Don't wait until the end of the talk!
- Wlodek will tell me when there is a question.

Where does all this come from

- I love physics.
- I love math.
- I asked my professor “Where does all this come from”.
- He said “We don’t know!”
- We still don’t know, but I will give some insights I have found since then.

Matter Particles circa 1930s

Particle	Mass (MeV)	Charge	Force	Size	Spin (h)
Proton p	938.3	+	S, E, W, G	10^{-15} m	1/2
Neutron n	939.6	0	S, W, G	10^{-15} m	1/2
Electron e	0.511	-	E, W, G	$< 10^{-20}$ m	1/2
Neutrino ν	$\approx 10^{-7}$	0	W, G	$< 10^{-20}$ m	1/2

$$\text{Spin} = h/2$$

- In 1920s, two young physicists could understand their data if the electron was spinning.
- Wrote a paper.
- Then the big boss came back from sabbatical.
- They tried to pull the paper, but it was already published.
- The reason the electron has spin $1/2$ is so that we could be here to discover

Gauge Symmetries circa 1970s

$$\text{SM} = \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$$

Force	Carrier	Spin
Strong	Gluon	1
Electro-Magnetic	Photon	1
Weak	W	1
Gravity	Graviton	2
Mass	H	0

Weak Force

- Free neutron decay half-life ~ 10 minutes. $n \rightarrow p e \bar{\nu}$
- If earth to sun filled with Pb, most neutrinos would still get through.
- Strong $N^* \rightarrow p \pi$ 10^{-22} sec
- Electromagnetic $\pi \rightarrow \gamma \gamma$ 10^{-18} sec
- Hydrogen $p e \rightarrow n \nu$
- Without the weak interaction, there would be no energy from the sun, no elements but Big Bang H, He.
- The strength of the weak force determines the lifetime of the sun.
- Why don't neutrons in nucleus decay?

Physics

- Binding energy of the deuteron = 2.2 MeV.
- If neutron were 0.1% heavier, deuteron would be radioactive!
- If proton were heavier than the neutron, hydrogen would be radioactive!
- Physics is exactly what it has to be in order for us to be here to discover the laws of physics.

Quantum Mechanics

- Developed 1910 – 1950 by:
- Niels Bohr – “Anyone who thinks they understand QM, and is not deeply disturbed by it, doesn’t understand QM.”
- Albert Einstein – “It’s spooky action at a distance”
- Erwin Schrödinger – “I wish I never discovered these damn wave functions.”
- I love QM! It is not trivial!
- Without QM we wouldn’t be here to discover QM.

Imaginary Numbers

- Two centuries ago mathematicians discovered the number $i \equiv \sqrt{-1}$. $i^2 = -1$.
- They called it imaginary because it couldn't describe anything that is "real".
- They worked out the beautiful mathematics.
- $e^{i\theta} = \cos \theta + i \sin \theta$.
- QM needs i !

Quantum Mechanics

- Electron is described by Schrödinger wavefunction: Ψ
- QM: Rotate by an angle θ : $\Psi' = e^{iS\theta}\Psi = (\cos(S\theta) + i \sin(S\theta))\Psi$
- $\theta = 360^\circ$, $\Psi' = \cos(2\pi S) \Psi$
- Spin 0 or 1, $\Psi' = \Psi$. Spin $\frac{1}{2}$, $\Psi' = -\Psi$.

QM

- Can't put two electrons in the same place at the same time.
- Spin-statistics symmetry, Chemistry,
- Spin $\frac{1}{2}$ are matter particles!
- Spin 0, 1, 2 are force particles!
- Very simple, but profound.

Magnetic Moment 1930s

- Particle is spinning,
- Particle is charged,
- Spinning charge creates a magnetic field:
- $\mu = \frac{gQS}{M}$
- Dirac Equation: $g = 2$ for a spin $\frac{1}{2}$ point particle.
- Proton: $g = 5.6$, finally explained by quark model.
- Electron: $g = 2.0$
- Oppenheimer et al. calculated the first order correction to 2 to be infinity.

Spin ½ Particles 1940s – 1980s

Three Generations!

Particle	Mass (MeV)	Particle	Mass (MeV)	Particle	Mass (MeV)
u	312	c	1750	t	171200
d	313	s	490	b	5620
e	0.5	$\mu \rightarrow e \nu \bar{\nu}$	105	τ	1777
ν_e	$\sim 10^{-7}$	ν_μ	$\sim 10^{-7}$	ν_τ	$\sim 10^{-7}$

More Symmetries

- C symmetry – changes particle to anti-particle.
- P symmetry – changes x to $-x$.
- T symmetry – changes t to $-t$.
- Good symmetries for strong, EM, gravity.
- Discovered in 1950 – 60s: P, T, C are broken symmetries in the weak interaction.
- Without weak interaction broken symmetries, we wouldn't be here.

Theory 1970s

- In SM we can only get this if there are three, or more, ways for a given reaction to go, and get QM interference with the three amplitudes, with at least one imaginary amplitude.
- Need at least three generations.
- In the big bang all the non-neutrino particles/anti-particles should have finally annihilated to photons.
- Due to symmetry breaking, $p/\text{photon} \approx 10^{-9}$.

Theory 1980s - 2022s

- Not enough symmetry violation in the Big Bang SM for us to be here.
- Maybe more symmetry violation in neutrino sector? – first two talks in the lecture series.
- Maybe super-symmetry? SUSY.
- Symmetry between spin $\frac{1}{2}$ and spin 1.
- Selectron is a force particle! Sphoton is a matter particle! Different SM/SUSY masses, so symmetry is broken.

Electron $a = \frac{g-2}{2} = 0.0011$

- 1948 I.I. Rabi, Conference at Shelter Island.
- Schwinger et al.,
- Quantum Electrodynamics (QED).
- “Renormalization” gets rid of the infinities.
- For any new theory, the first question: “Is it renormalizable”.

QED Renormalization

- **Renormalization** is a collection of techniques in [quantum field theory](#), the [statistical mechanics](#) of fields, and the theory of [self-similar](#) geometric structures, that are used to treat [infinities](#) arising in calculated quantities by altering values of these quantities to compensate for effects of their **self-interactions**.
- For example, an [electron](#) theory may begin by postulating an electron with an initial mass and charge. In [quantum field theory](#) a cloud of [virtual particles](#), such as [photons](#), [positrons](#), and others surrounds and interacts with the initial electron. Accounting for the interactions of the surrounding particles (e.g. collisions at different energies) shows that the electron-system behaves as if it had a different mass and charge than initially postulated. Renormalization, in this example, mathematically replaces the initially postulated mass and charge of an electron with the experimentally observed mass and charge.

Quantum Mechanics

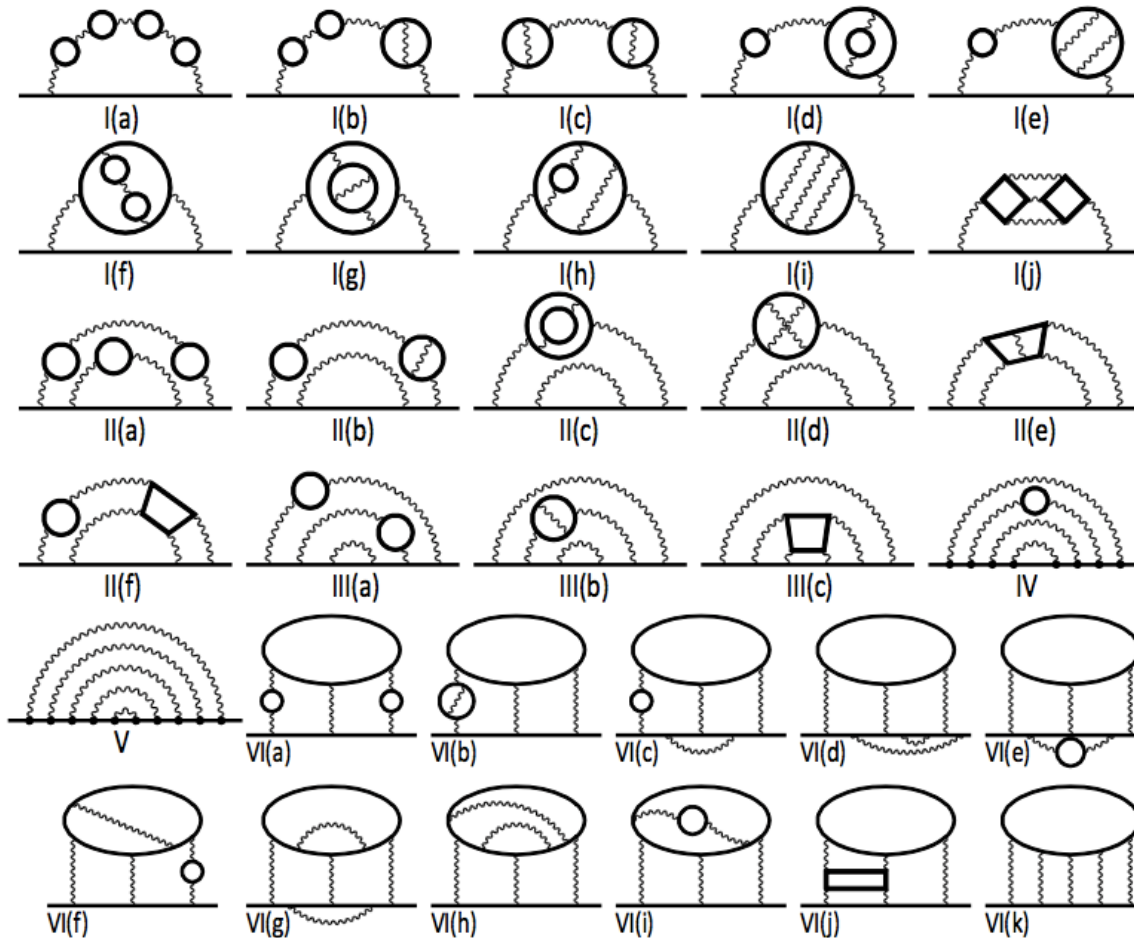
- Anomalous magnetic moment is due to QM.
- The energy of the vacuum classically is zero.
- QM: $dE \times dt = h$
- The problem with zero is that it has no uncertainty.

Quantum Electrodynamics

Not Boring!

10th
12672
diagrams

$$dE \times dt = h$$



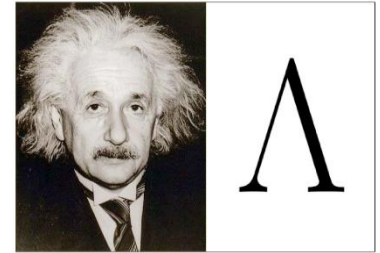
**What about
Einstein's
General
Theory
Relativity**

History of the universe

- Big bang (birth of our space-time) 14B years ago.
- I was born.
- We figured out that the big bang happened 14B years ago.
- Georges Lemaître, (1894-1966), Belgian cosmologist, Catholic priest, and father of the Big Bang theory.
- That the entire observable universe of galaxies began with a bang seemed preposterous.

Einsteins' Cosmological Constant

- GR has cosmological constant.
- Einstein at first had static universe.
- Chose value of Λ to stop the universe from collapsing.
- Then Hubble et al. found that the universe was expanding!
- No need for cosmological constant.
- Einstein: “Biggest blunder of my life”.



“Dark Energy”

- Theorists calculated QM of the vacuum in 1960s: $\Lambda \approx 10^{120}$.
- Steve Weinberg: need $\Lambda < 2$ for us to exist.
- “Dark energy” discovered in 1990s: $\Lambda = 0.7$.
- Total energy density = 1.0.
- aka cosmological constant.
- “Worst theoretical embarrassment in the history of physics” – Ed Witten.
- A new symmetry, such as SUSY, would reduce it,

Future of our universe

- For a matter dominated universe, the pull of gravity slows the expansion rate vs. time.
- For a dark energy dominated universe, the expansion rate increases with time.
- We now live in a dark energy dominated universe.
- Big bang was the creation of space-time.
- Faster than the speed of light? Yes.

Meta-stable? Quantum tunneling?

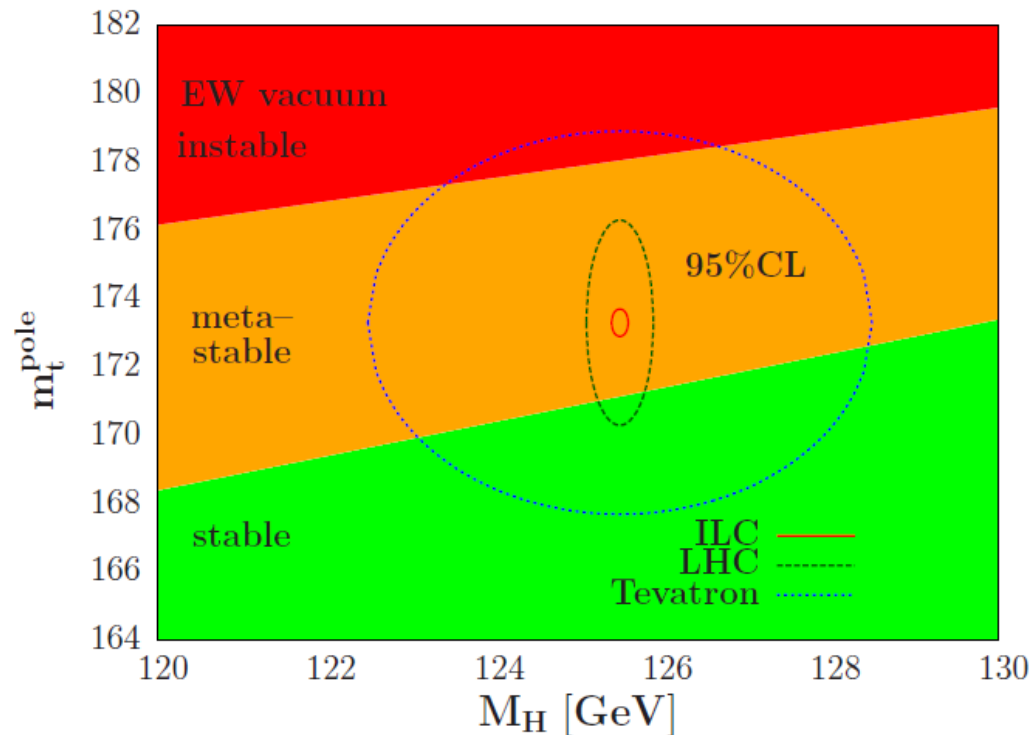


Figure 1: The 2σ ellipses in the $[M_H, m_t^{\text{pole}}]$ plane that one obtains from the current top quark and Higgs mass measurements at the Tevatron and LHC and which can be expected in future measurements at the LHC and at the ILC, when confronted with the areas in which the SM vacuum is absolutely stable, metastable and unstable up to the Planck scale.

Quantum Mechanics

- All particles exist in the vacuum.
- All particles contribute to the anomalous magnetic moment.
- Are there new particles beyond the SM?
- Are there new symmetries?
- E821 measure anomalous magnetic moment to 0.5 parts per million (ppm).
- Theorists: calculate to 0.5ppm. Compare.

BNL E821 Experiment

- Accelerate protons through 24 billion volts.
- Electrical outlet = 110V.
- Energy = 24 billion eV (GeV).
- Protons hit target.
- $E = mc^2$
- Convert energy into mass
- $P + P \rightarrow$ muons, anti-muons, electrons, anti-electrons, Mini-big bang.
- Muon $m = 0.105 GeV$.
- Beamline selects particles.

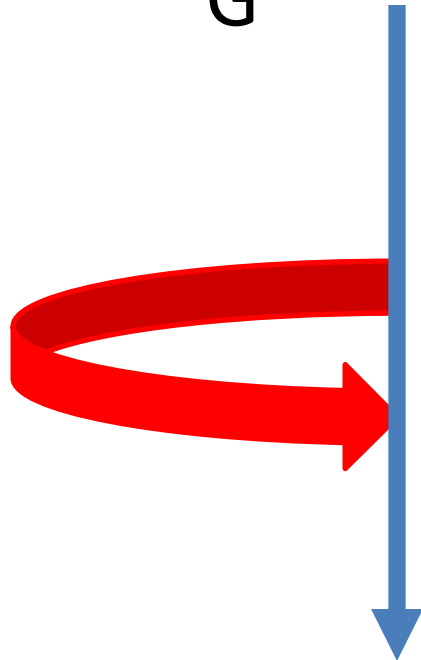
Spin Precession

spinning top

G

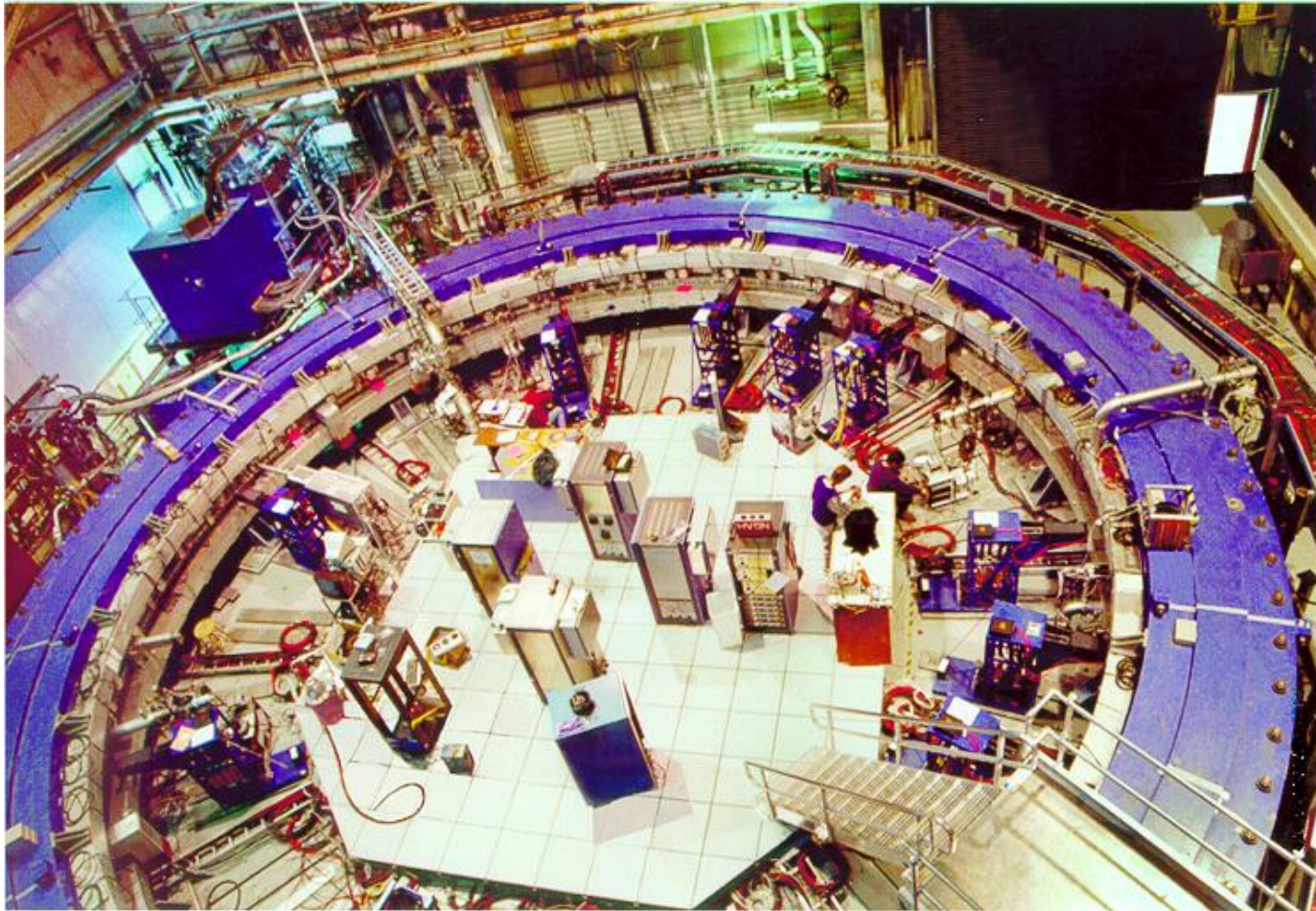
μ magnetic moment

B



$$\omega_a = \frac{eaB}{m}$$

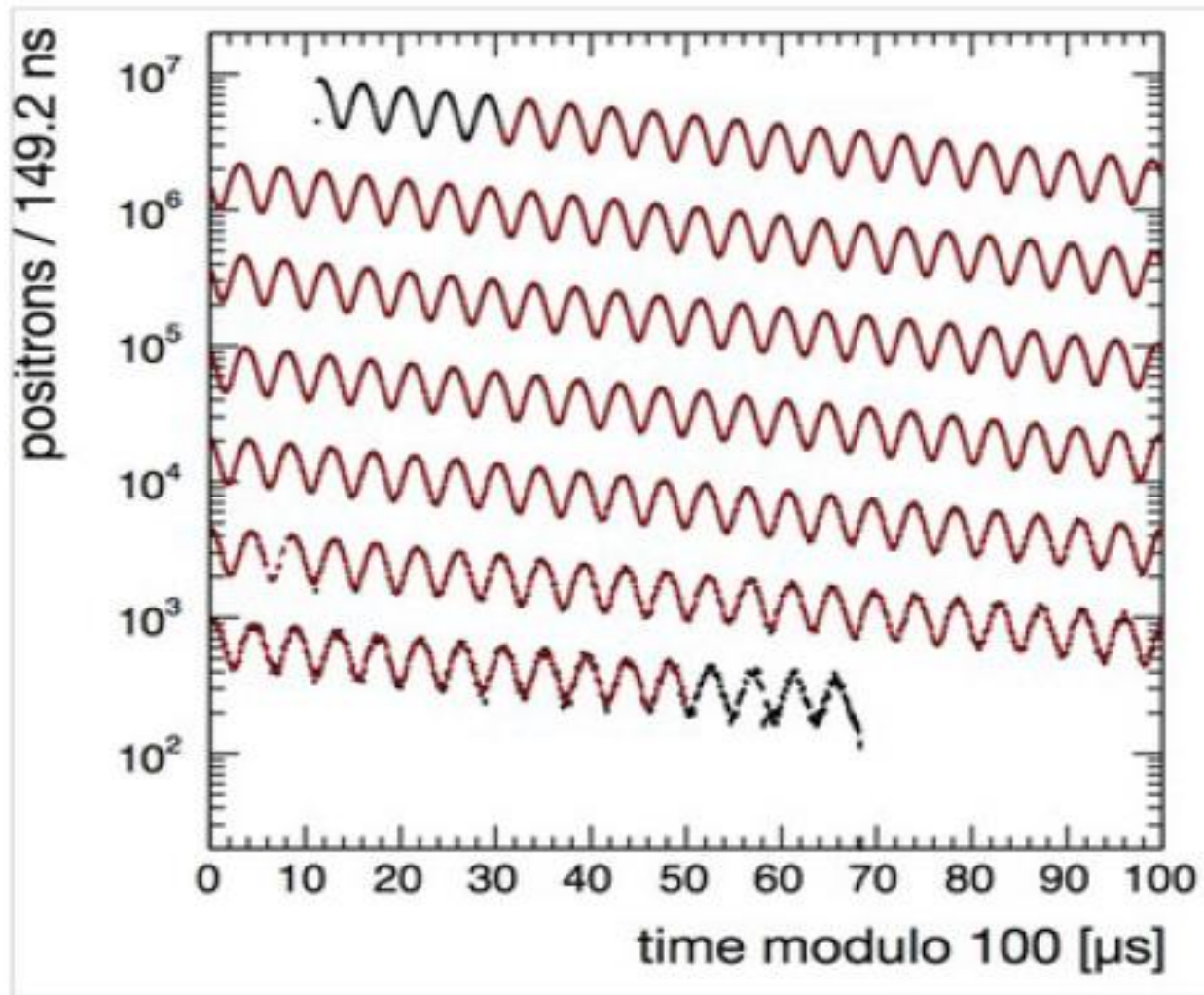
Storage Ring Magnet at BNL



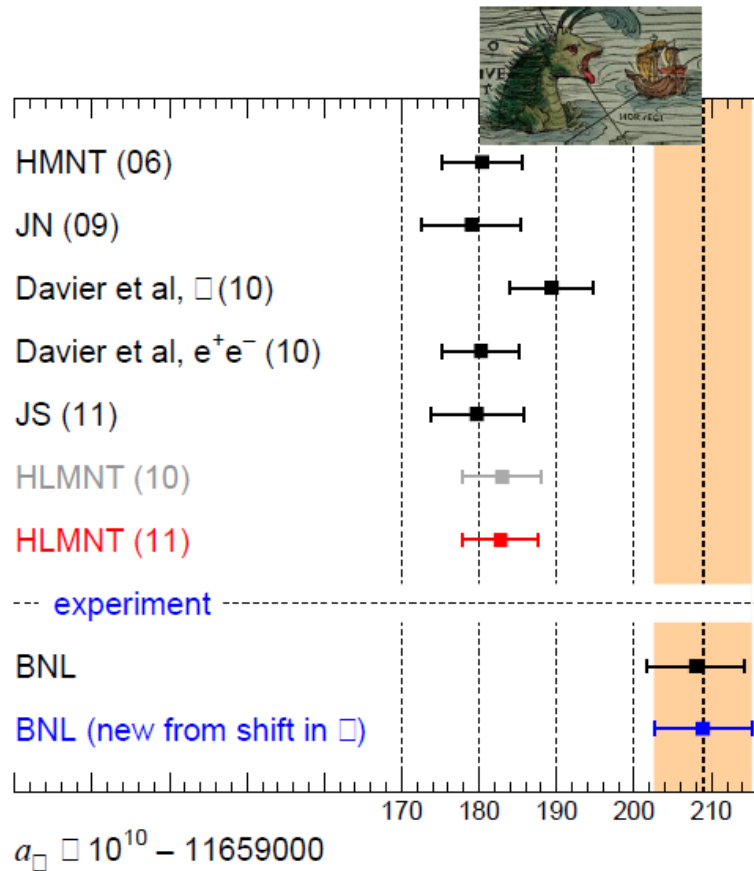
$$\mu \rightarrow e \nu \bar{\nu}$$

- Weak decay.
- The high energy positrons go preferentially in the direction of the muon spin.
- Just measure the high energy positrons.

High Energy Positrons vs. Decay Time



BNL E821



- The last a_μ experiment ended at BNL in 2001
- 3σ discrepancy with the theoretical expectation
- Goal of FNAL g-2 is to reduce the experimental error by a factor of 4

Theoretical evaluations

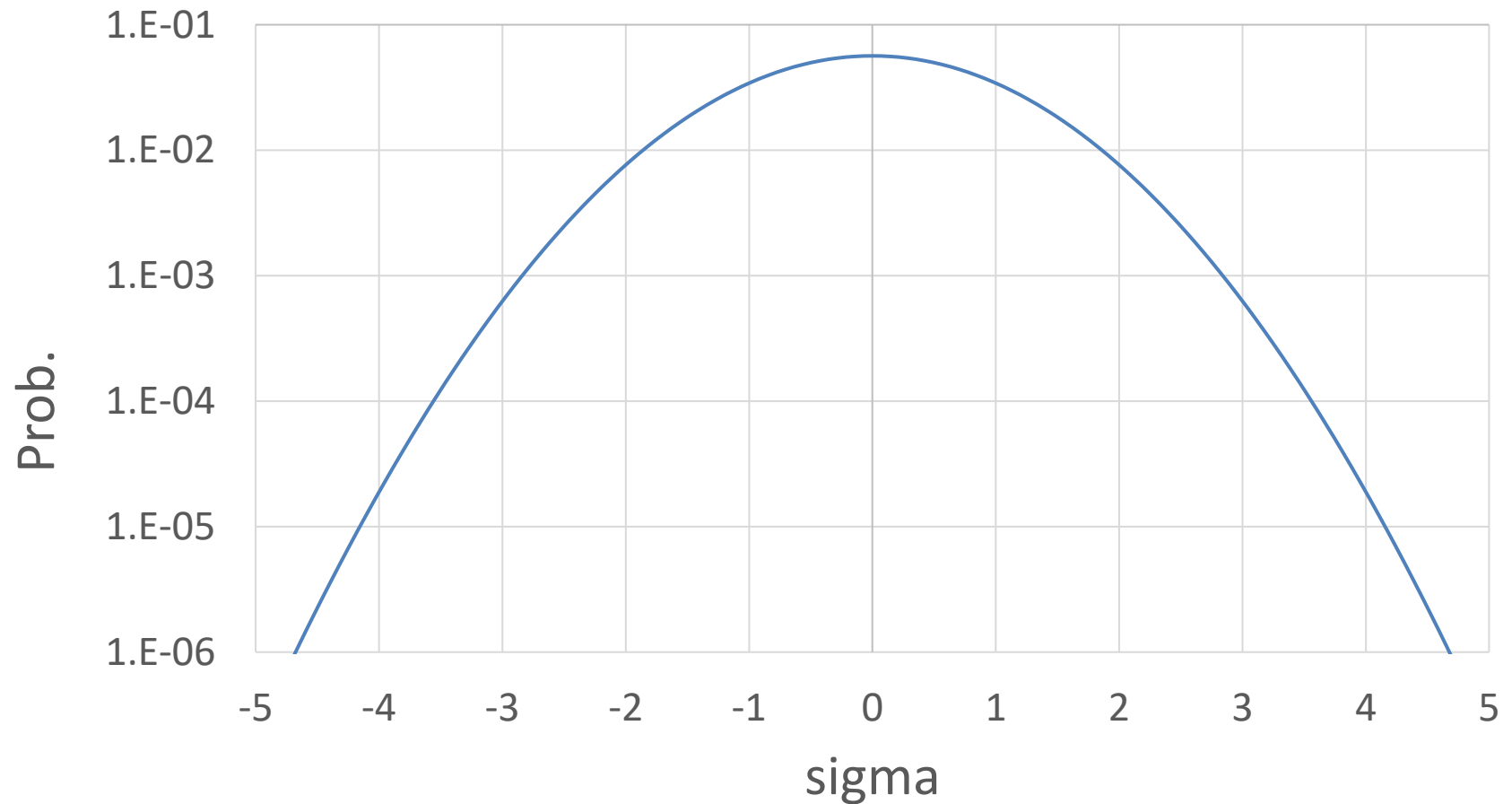
Last experimental result from BNL

What is 3σ ?

- Probability $\approx e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$
- Toss a coin 100 times.
- How many heads? $\bar{x} = 50$.
- $\sigma = \sqrt{50}$

0	1	2	3	4
50	43	36	29	21

$$\text{Probability} \approx e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$



FNAL

- FNAL is a National Lab,
- 1 hour west of Chicago, IL.
- They were on the “energy frontier”.
- Center of mass energy 2 TeV.
- CERN in Geneva, Switzerland, LHC 13 TeV.
- FNAL went from the energy frontier to the intensity frontier.
- $20 \times$ more muons. Move the magnet to FNAL.

Move from BNL to FNAL



Smith Point Marina



Around FL, and up Miss. River



William Morse - BNL

Up Illinois River



Entrance Ramp to Interstate 88



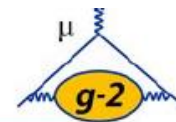
Interstate 88



Arriving at FNAL 2013



Ring reassembly at Fermilab

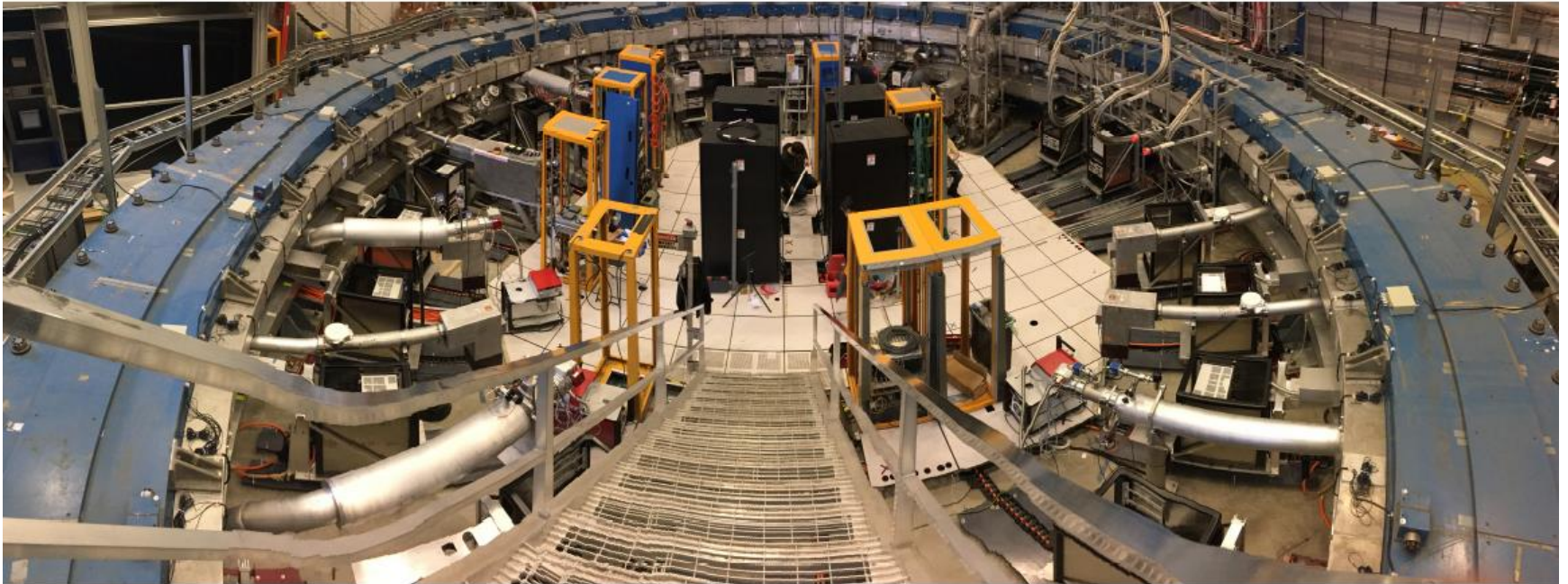


2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021



June 23, 2014. Bottom yoke. Reassembly progresses well. Superconducting coils will be moved into the experimental hall end of July 2014

FNAL 2017



To Be Continued

- FNAL is now churning out data.
- Theorists are still calculating with super-computers.
- Talk by Vladimir Tishchenko.